Lecture #0.2

Control systems requirements

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Overview

- Steady-state requirements
  - asymptotic stability
  - steady-state error

- Transient requirements
  - step response

- Application to PID control

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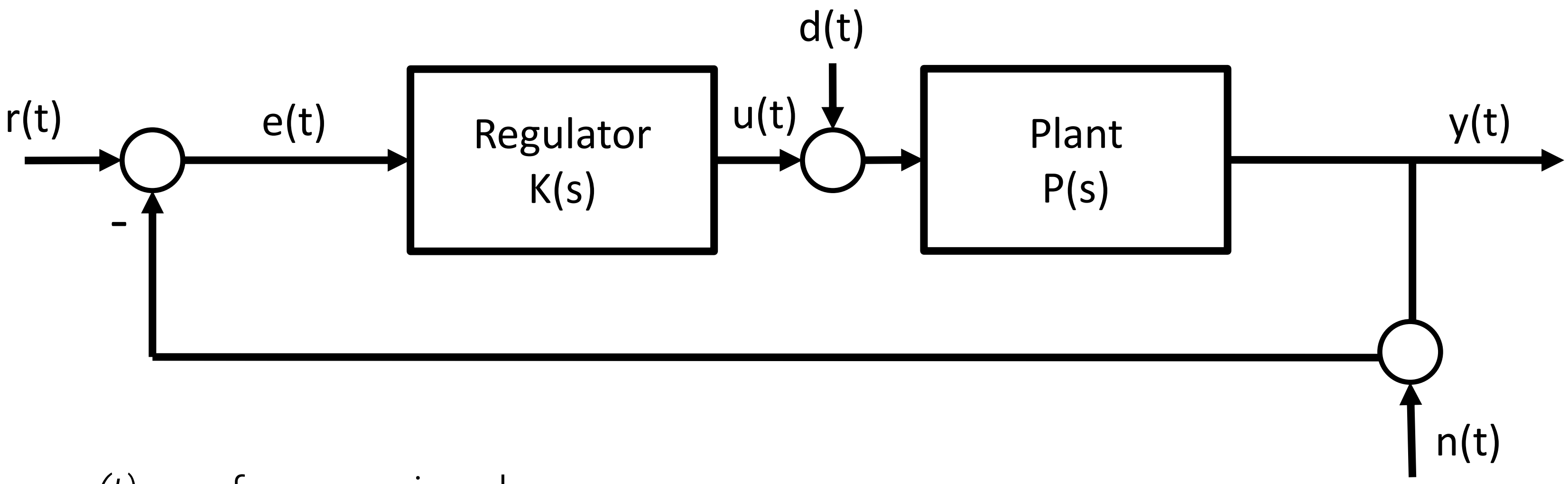
Reference scheme



- *r(t)* – reference signal
- *e(t)* – control error
- *u(t)* – control action
- *y(t)* – controlled output
- *n(t)* – measurement noise
- *d(t)*– additive (control action) disturbance

Control systems specifications

- When designing a control system, some **design requirements** must be met

- Typically
  - **Steady-state** error
  - **Transient** performance
  - **Noise** and **disturbance** rejection
  - **Robustness**

Control systems specifications

- For simplicity, we will consider SISO plants
- **Linearity** allows to easily deal with additive noise and disturbance signals
- All the requirements must be met also in presence of uncertainties

Asymptotic stability

- First basic requirement:
  → closed-loop asymptotic stability
  - Hurwitz criterion
  - Nyquist criterion

- Sometimes other stability properties are considered
  (e.g. Bounded-Input-Bounded-Output)

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Response to standard signals

- Further requirements on **static and dynamic** performance are usually given with reference to the response to **standard signals and disturbs**

- Often **polynomial** signals are considered (step, ramp, parabola…)

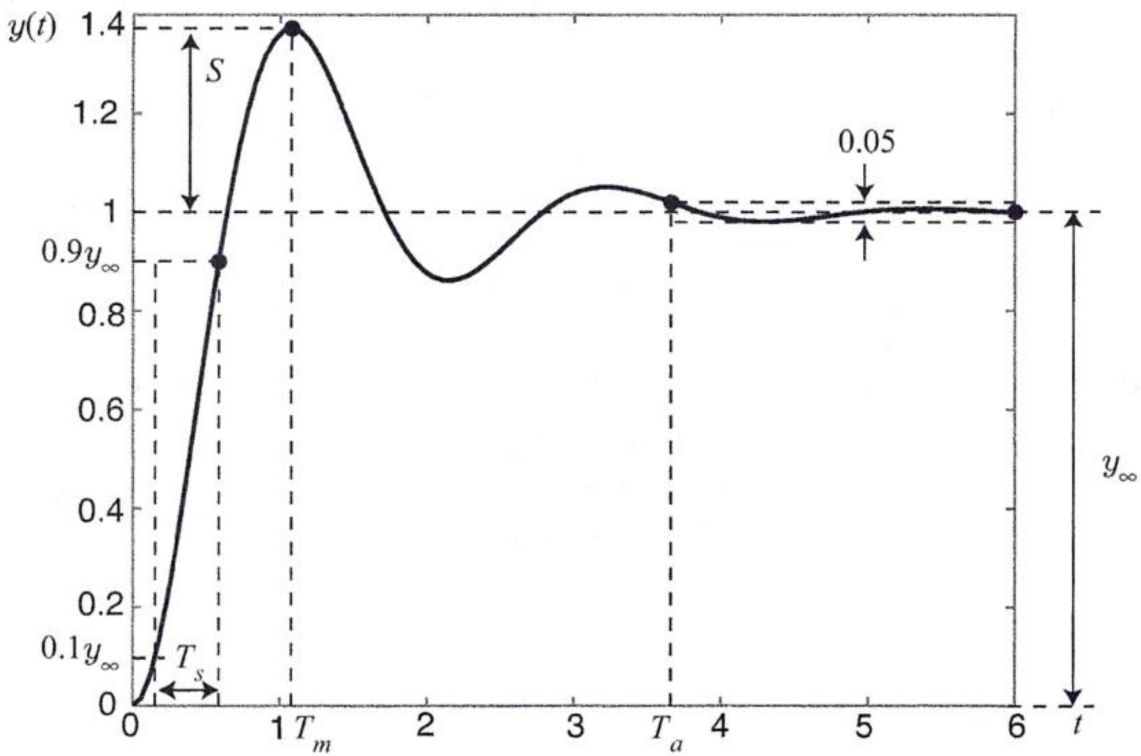
Response to standard signals

- The response can be divided into
  - Transient ↔ dinamic specifications
  - Steady-state ↔ static specifications

- Often given in terms of the system's step response

Step response

- With respect to the step response, we can define:
  - Steady-state value
  - Rising time
  - Settling time
  - Maximum overshoot
  - Time of maximum overshoot



Step response

- Steady-state value: value of the output after the end of the transient (y_∞)
- Rising time: time needed to go from 10% to 90% of y_∞ (T_s)
- Settling time: time interval after which the output remains in a given range around y_∞, usually $\pm5\%$ or $\pm1\%$ ($T_{a,\varepsilon\%}$)
- Maximum overshoot: maximum distance of the output from y_∞

$$S = \max\left(\frac{y - y_\infty}{y_\infty}\right)$$

- Time of maximum overshoot: time instant when the maximum overshoot is reached (T_m)

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Control requirements

- Typically require that the error at steady-state is smaller than a given maximum value

$$e_\infty = \lim_{t \to \infty} |r(t) - y(t)| \le e_{max}$$

- Transient requirements usually assign an upper limit to T_a, T_s, S

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Control *action* requirements

- In addition, **physical limitation on the control action** must also be considered:
  - **maximum admissible value** of the control
  - **energetic considerations:** energy consumption, components overheating, etc.

Harmonic response function

- In practice, we often want to study properties of the closed-loop transfer function

$$F(s) = \frac{P(s)K(s)}{1 + P(s)K(s)}$$

- by looking at properties of the loop function

$$L(s) = P(s)K(s)$$

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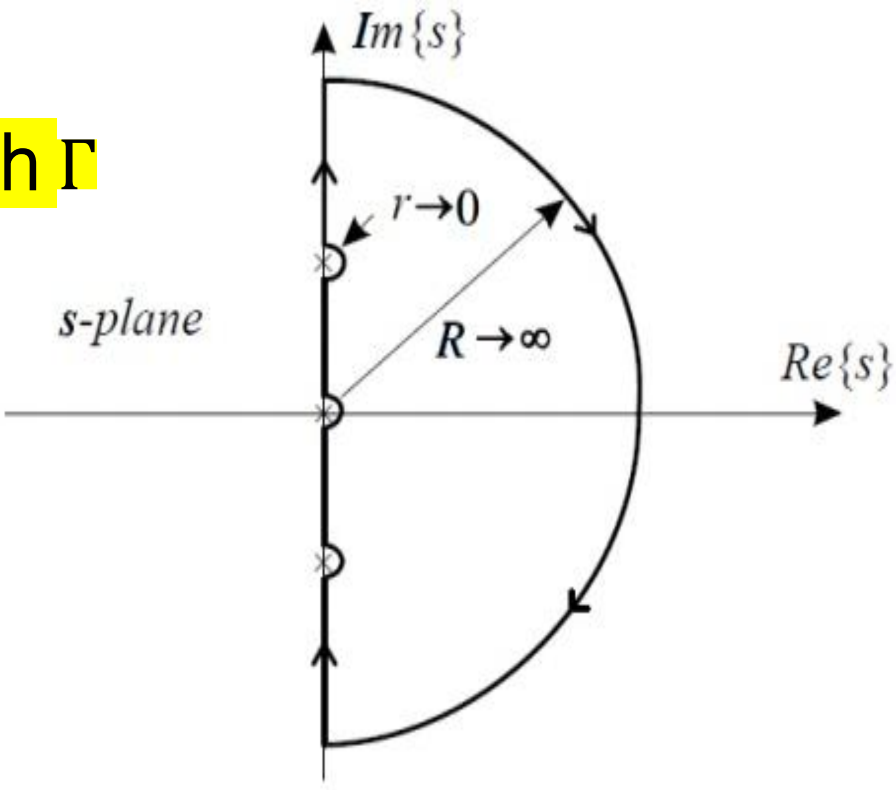
Nyquist criterion

- We want to check the number of RHP zeros of

$$den(s) = 1 + L(s)$$

- Consider the Nyquist path Γ

→ all frequencies
+
the point at infinity
(encircling the RHP)



Nyquist criterion

- From Cauchy's argument principle

logarithmic derivative
extracts the argument of $e^{i\theta}$

number of time
Γ encircles the origin

zeros of D(s)
enclosed by Γ

$$N = -\frac{1}{2\pi i}\int_{\Gamma} \frac{den'(s)}{den(s)}\,ds = (Z - P)$$

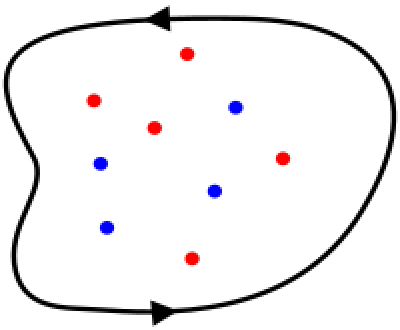
CW = positive
CCW = negative

a full encirclement

poles of D(s)
enclosed by Γ

- zeros of $den(s)$ = closed-loop poles

$$\rightarrow \boldsymbol{Z = 0 \Leftrightarrow N + P = 0}$$



Nyquist criterion

- To have **# CL RHP poles = 0**

# CCW windings of $1 + L(\Gamma)$ around the origin	=	**# OL RHP poles**

- typically we look at the Nyquist diagram of $L(\Gamma)$

→ look at the encirclements around -1

Stability margins

- If the system is stable, we want to *stay away* from the critical point

$$-1 + i0 = e^{i\pi}$$

  - avoid magnitude 1 at phase $-\pi$
  - avoid phase $-\pi$ at magnitude 1

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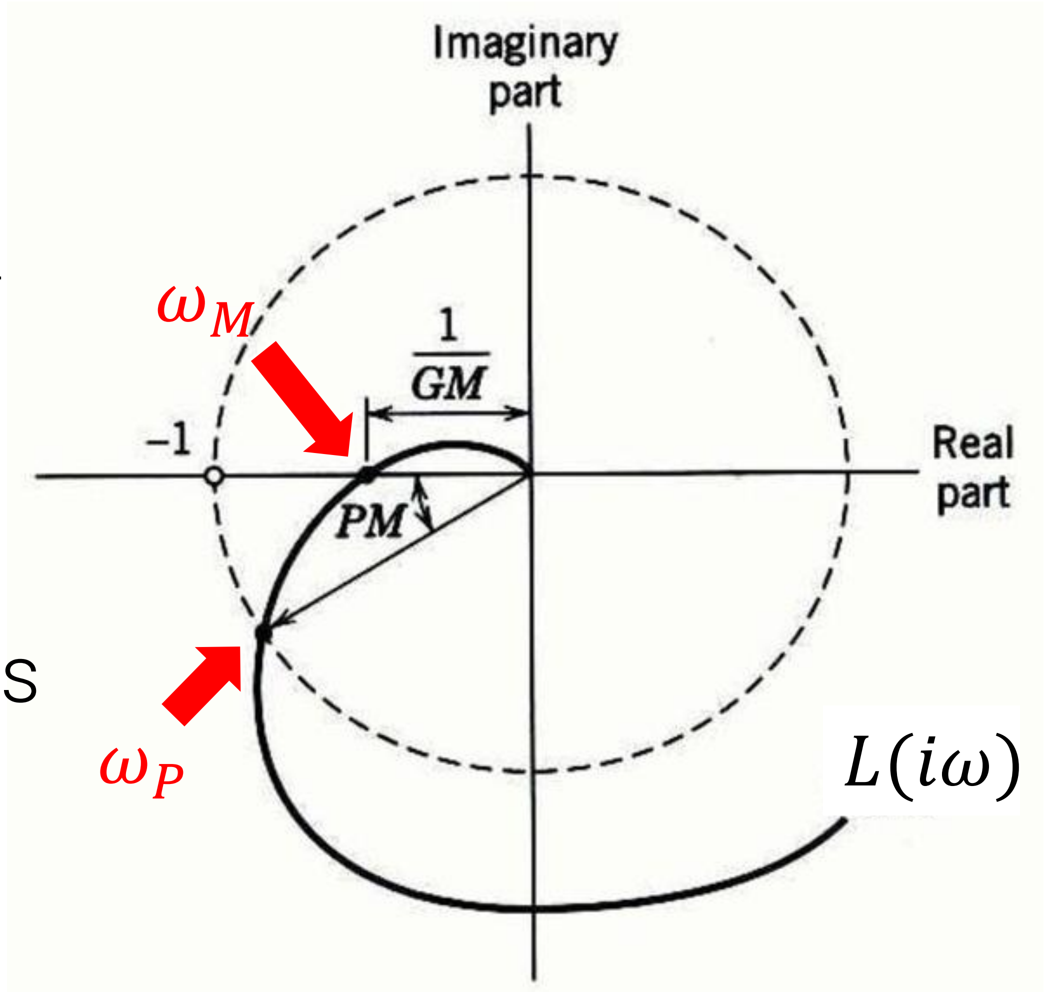
Stability margins

- Gain margin:
  amplification that brings $|L(i\omega_M)|$ to 1 for $\measuredangle L(i\omega_M) = -\pi$

- Phase margin:
  phase lag that brings $\measuredangle L(i\omega_P)$ to $-\pi$ for $|L(i\omega_P)| = 1$



More requirements!

- For a good control system, we want
  - $Y(s) \approx R(s)$
  - $U(s)$ small

- But we also want to reject external signals
  - $R(s) \rightarrow Y(s), U(s)$
  - $D(s) \rightarrow Y(s), U(s)$
  - $N(s) \rightarrow Y(s), U(s)$

Loop shaping

- In practice, for LTI SISO systems, all these requirements can be translated in terms of **constraints on the (open-loop) harmonic response function** of the series controller-plant

→ **loop shaping**

Loop shaping

- Moreover (without going into details…):
  - **settling time, disturbance and noise rejection** are linked to the system's **bandwidth**
  - **overshoot** is linked to **phase margin**
  - **steady-state** error is linked to **dc-gain**

$R(s) \rightarrow Y(s)$

- $Y(s) = P(s)K(s)E(s) = P(s)K(s)\big(R(s) - Y(s)\big)$

  (we neglect n and d here, remember superposition principle…)

$$\Rightarrow Y(s) = \underbrace{\frac{P(s)K(s)}{1 + P(s)K(s)}}R(s)$$

$F(s)$
Complementary sensitivity:
we want $F(s) \approx 1$

$D(s), N(s) \rightarrow Y(s)$

- $Y(s) = -P(s)K(s)\big(Y(s) + N(s)\big)$

$$\Rightarrow Y(s) = -F(s)N(s)$$

We want $F(s) \approx 0$

- $Y(s) = -P(s)K(s)Y(s) + P(s)D(s)$

$$\Rightarrow Y(s) = \frac{P(s)}{1 + P(s)K(s)} D(s)$$

Process sensitivity
we want $L(s) \rightarrow \infty$

Loop shaping